Solve the differential equation $dy/dx = x \ln(x)$ subject to the initial condition $y = -1$ when $x = 1$. Confirm the solution graphically by showing that it conforms to the slope of the field.

Differential Equations

$$\frac{dy}{dx} = x^2 + 7$$

$$\int dy = \int x^2 + 7 \, dx$$

$$y = \frac{x^3}{3} + 7x + C$$

General Solution.

y = 4 when x = 1

particular Solution.

$$4 = \frac{1}{3} + 7 + C$$

$$-\frac{10}{3} = C$$

$$y = \frac{x^3}{3} + 7x - \frac{10}{3}$$

$$\frac{dy}{dx} = 2x^2 - 4$$

~~**Separation of variables...**~~

$$\frac{dy}{dx} =$$

$$y = 2e^{x} - \sin x + C$$

$$3 = 2(1) - \sin 0 + C$$

$$3 = 2 - 0 + C$$

$$1 = C$$

$$\frac{dy}{dx} = \frac{x}{y} \quad \text{initial condition } (0,3)$$

$\cancel{\sqrt{9+x^2}}$

$$dy = \frac{x}{y}\,dx$$

$$\int y\,dy = \int x\,dx$$

$$\frac{y^2}{2} = \frac{x^2}{2} + C$$

$$y^2 = x^2 + C$$

$$9 = C$$

$$\frac{y^2}{2} = \frac{x^2}{2} + \frac{9}{2}$$

$$y^2 = x^2 + 9$$

$$y = \pm\sqrt{x^2+9}$$

$$y = \sqrt{x^2+9}$$

$$\frac{dy}{dx} = (y + 2)(2x - 5) \qquad y = 3 \text{ when } x = 0$$

$$dy = (y+2)(2x-5)\,dx$$

$$\int \frac{1}{y+2}\,dy = \int (2x-5)\,dx$$

$$|y+2| = Ce^{x^2-5x}$$

$$\ln|y+2| = x^2 - 5x + C$$

$$|y+2| = e^{x^2-5x+C}$$

$$|y+2| = e^{x^2-5x}e^{C}$$

$$\frac{dy}{dx} = \frac{y+2}{x-5} \qquad (2,4)$$

$$dy = \frac{y+2}{x-5}\,dx$$

$$\int \frac{1}{y+2}\,dy = \int \frac{1}{x-5}\,dx$$

$$\ln|y+2| = \ln|x-5| + \ln 2$$

$$\ln|y+2| = \ln 2|x-5|$$

$$y+2 = \pm 2(x-5)$$

$$\frac{dy}{dt} = 3t^2 \cos(t^3)$$

$$\int dy = \int 3t^2 \cos(t^3)\, dt.$$

$u = t^3$

$du = 3t^2\, dt.$

$$y = \int \cos u \; du$$

$$\boxed{= \sin t^3 + C}$$

$$\frac{dy}{dx} = 5\sec^2 x - (3/2)\sqrt{x} \qquad y = 7 \text{ when } x = 0$$

$$\frac{dy}{dx} = \sin(x^2) \qquad (1,5)$$

Section 6.4

Exponential Growth and Decay

Seperable Differential Equation

A differential equation of the form dy/dx = f(y)g(x) is called seperable.

$$\frac{1}{f(y)}\, dy = g(x)\, dx$$

The solution is found by antidifferentiating each side with respect to its isolated variable

Solve for y if $dy/dx = (xy)^2$ and $y = 1$ when $x = 1$.

The Law of Exponential Change

If y changes at a rate proportional to the amount present (that is, if $dy/dt = ky$), and if $y = y_0$, then

$$y = y_0e^{kt}$$

The constant k is the **growth constant** if

$$k > 0$$

or the **decay constant** if $k < 0$

Show $y = y_0e^{kt}$ can be found by solving for y in the differential equation :

$$\frac{dy}{dt} = ky$$

Continuously Compounded Interest

If A_0 dollars are invested at a fixed annual interest rate r.

If interest is added to the account k times a year,

The amount of money present after t years is...

$$A(t) = A_0\left(1 + \frac{r}{k}\right)^{kt} \text{ (NOT continuously)}$$

The interest added (COMPOUNDED) monthly (k = 12), weekly (k = 52), daily (k = 365)...

$$A(t) = A_0\left(1 + \frac{r}{k}\right)^{kt}$$

Differential Equation : $\dfrac{dA}{dt} = rA$

Initial Condition : $A(0) = A_0$

Amount of money in account after t years is

$A(t) = A_0 e^{rt}$ Continuously

Interest paid according to this formula is compounded continuously.

Suppose you deposit $800 in an account that pays 6.3% annual interest. How much will you have 8 years later if the interest is

a.) compounded continuously?
b.) compounded quarterly?

Suppose you deposit $900 in an account at 5.5% annual interest. How much will you have 7 years later if compounded continuously and compounded quarterly?

Radioactivity

If y_0 is the number of radioactive nuclei present at time zero, the number still present at any later time t will be

$$y = y_0 e^{-kt}, \ k > 0$$

The half-life of a radioactive element is the time required for half of the radioactive nuclei present in a sample to decay.

Find the half-life of a radioactive substance with decay equation $y = y_0e^{-kt}$ and show that the half-life depends only on k.

Half-life

The half-life of a radioactive substance with rate constant k (k > 0) is

$$\text{half-life} = \frac{\ln 2}{k}$$